The First Order Density Corrections Including Ternary Collisions in a Boltzmann-Landau Gas

ARUN K. MITRA

Department of Mathematics

and SUNANDA MITRA

Department of Electrical Engineering, Texas Technological University, Lubbock, Texas

(Z. Naturforsch. **25 a**, 121—126 [1970]; received 20 October 1969)

The first order density corrections to the coefficients of viscosity (η) and thermal conductivity (λ) in a Boltzmann-Landau gas have been calculated including the effect of ternary collisions.

1. Introduction

The aim of this paper is to give a method of formal expansions in powers of density of the coefficients of thermal conductivity (λ) and viscosity (η) of a moderately dense Boltzmann-Landau gas, based on the assumptions given in a previous paper [1]. We shall consider here particularly the contributions of the ternary collision process to the transport integral equations. This will describe completely the first order density correction to the transport coefficients (the quadruple process does not contribute to the first order density correction). The general integral equation governing these transport phenomena in the first order Chapman-Enskog equation including multicollision processes will be assumed as given by [(2.2a) [1] and (3.14) [2]]:

$$\begin{aligned} L(X) &= \int \sigma(f_0)\, f_0(p)\, f_0(p_1)\, [X(p_1') + X(p') - X(p_1) - X(p)]\, \mathrm{d}p_1'\, \mathrm{d}p_1\, \mathrm{d}p' \qquad (1.1) \\ &= f_0(p) \left[\frac{1}{T} \frac{\partial T}{\partial x_i}\, p_i\, R(p) + A_{ij}\, S_{ij}(p) \right] \end{aligned}$$

subject to the subsidiary condition [(2.7a), [1]]. First of all, we shall see that the pressure tensor $\pi_{\mu\nu}$ and heat flow Q can be expressed, in this first order Chapman-Enskog approximation in terms of $X(p)$ instead of φ in ref. [1]. This might seem useful, since (1.1) is a symmetric integral equation. The general expressions of $\pi_{\mu\nu}$ and Q will not be deduced here. Instead we shall use the known results given earlier by GROSSMANN [2–4].

2 A. The Pressure Tensor $\Pi_{\mu\nu'}$ $(\mu \neq \nu)$

The pressure tensor for any arbitrary distribution f is given in [2–4] to be

$$\pi_{\mu\nu} = \int (P_\mu - m\, u_\mu) \left(\frac{\partial \varepsilon_p(f)}{P_\nu} - u_\nu \right) f(P)\, \mathrm{d}P \qquad (2.1)$$

for $\mu \neq \nu$.

Here $u = \langle P/m \rangle_f = \langle \partial\varepsilon/\partial P \rangle_f$ is interpreted as the mean velocity. For the first order Chapman-Enskog approximation we have demanded [5] (as our subsidiary condition):

$$\langle P/m \rangle_f = \langle P/m \rangle_{f_0},$$

where $f = f_0 + f_1$, $f_1 \ll f_0$.

According to [1], (2.2c) we can put in the first order in f_1,

$$\varepsilon_p(f) = \varepsilon_p(f_0) + \int \widetilde{F}(P, Q; f_0)\, f_1(Q)\, \mathrm{d}Q\,. \qquad (2.2)$$

Since, for $f = f_0$, the non-diagonal elements of $\pi_{\mu\nu}$ vanish, we have by (2.1) and (2.2), in the first order in f_1:

$$\begin{aligned} \pi_{\mu\nu} = &\int p_\mu \left(\frac{\partial \varepsilon_p(f_0)}{\partial P_\nu} - u_\nu \right) f_1(P)\, \mathrm{d}P \\ &+ \int p_\mu \frac{\partial}{\partial P_\nu} \widetilde{F}(P, Q, f_0)\, f_1(Q)\, f_0(P)\, \mathrm{d}P\, \mathrm{d}Q \end{aligned} \qquad (2.3)$$

where $p = P - m\,u$.

(2.3) is similar to that given by GROSSMANN [4] with the exception that $F(P,Q)$ in [4] is replaced by $\widetilde{F}(P,Q;f_0)$ to include multicollision processes. In order to express (2.3) in terms of $X(P)$, we recall,

Reprint requests to Dr. A. K. MITRA, Department of Mathematics, Texas Techn. University, *Lubbock/Texas 79 409*, USA.

[1] A. K. MITRA and SUNANDA MITRA, Z. Naturforsch. **25 a**, 115 [1970].

[2] S. GROSSMANN, Z. Phys. **182**, 24 [1964].

[3] S. GROSSMANN, Nuovo Cimento **37**, G 98 [1964].

[4] S. GROSSMANN, Z. Naturforsch. **20 a**, 861 [1965].

[5] A. K. MITRA and SUNANDA MITRA, The Boltzmann-Landau Transport Equation I, Proc. Cambridge Phil. Soc. **64**, 177 [1968].

Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung") beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition "no derivative works"). This is to allow reuse in the area of future scientific usage.

according [1], (2.9a):

$$\left(u_\mu - \frac{\partial \varepsilon_p(f_0)}{\partial P_\mu}\right) f_0(P) = \frac{1}{\beta}\frac{\partial f_0}{\partial P_\mu}. \qquad (2.4)$$

Then with $f_1 = f_0\,\varphi$, (2.3) becomes

$$\pi_{\mu\nu} = -\frac{1}{\beta}\int p_\mu \frac{\partial f_0}{\partial P_\nu}\varphi(P)\,dP + \int p_\mu \frac{\partial}{\partial P_\nu}\widetilde{F}(P,Q;f_0)\,f_1(Q)\,f_0(P)\,dP\,dQ.$$

Integrating the second term partially w.r.t. to p_ν and collective terms, we get

$$\pi_{\mu\nu} = -\frac{1}{\beta}\int p_\mu \frac{\partial f_0}{\partial P_\nu} X(P)\,dP \qquad (2.5)$$

where we have used

$$X(P) = \varphi(P) + \beta\int \widetilde{F}(P,Q;f_0)\,f_1(Q)\,dQ \qquad (2.5')$$

according to [1], (2.2b). Now under the restrictions on $F_\nu(P_1,\ldots,P_\nu)$ used in ref. [1], it is clear from (1.1) that $X(P)$ depends on P only through $p = P - m\,u$. Also using (2.4) we find

$$\frac{m}{\beta}\frac{\partial f_0}{\partial P_\nu} = -p_\nu\left(1 + \frac{m\,n}{p}\frac{\partial \widetilde{F}(p;f_0)}{\partial p}\right) f_0(p) = -S_1(p)\,f_0(p)\,p_\nu. \qquad (2.6)$$

Thus the pressure tensor is, for $\mu \neq \nu$,

$$\pi_{\mu\nu} = \frac{1}{m}\int p_\mu\,p_\nu\,S_1(p)\,f_0(p)\,X(p)\,dP. \qquad (2.7)$$

2 B. The Heat Flow Q

In the general case of arbitrary f the heat flow Q is defined by [2]:

$$Q_i = \int \varepsilon_p(f)\,\frac{\partial \varepsilon_p(f)}{\partial p_i}\,f(p)\,dp \qquad (2.8)$$

where we are considering the physical situation $u=0$ substituing (2.2) and $f = f_0 + f_1$, $f_1 = f_0\,\varphi$ in (2.8), Q can be put in the following form (in the first order in f_1)

$$Q_i = \int \varepsilon_p(f_0)\,\frac{\partial \varepsilon_p(f_0)}{\partial p_i}\,f_0(p)\,\varphi(p)\,dp - \int \varepsilon_p(f_0)\,\widetilde{F}(p,q;f_0)\,\frac{\partial f_0(p)}{\partial p_i}\,f_1(q)\,dp\,dq. \qquad (2.9)$$

Also, since in this case

$$\frac{1}{f_0}\frac{\partial f_0}{\partial p_i} = -\beta\frac{\partial \varepsilon_p(f_0)}{\partial p_i},$$

it turns out that

$$Q_i = -\frac{1}{\beta}\int \varepsilon_p(f_0)\,\frac{\partial f_0}{\partial p_i}\,X(p)\,dp \qquad (2.10)$$

where we have used (2.5′) to obtain (2.10).

Finally, using (2.6), we have

$$Q_i = \frac{1}{m}\int p_i\,\varepsilon_p(f_0)\,S_1(p)\,f_0(p)\,X(p)\,dp. \qquad (2.11)$$

The results (2.7) and (2.11) include multi-collision processes in the Boltzmann-Landau gas and take the form discussed in Ref. [6] in case of the binary collision approximation. To find $\pi_{\mu\nu}$ and Q and therefore the coefficients η and λ of viscosity and thermal conductivity respectively, we need to find the formal solution for $X(P)$ in the symmetric linear integral equation, which we shall now carry out.

3. The Formal Solution of the Transport Equation

Equation (1.1) looks similar to the ordinary Chapmann-Enskog equation for a Boltzmann gas. The solutions of the homogeneous equation are, in our case, the five collision invariants $\psi_i = 1$, p_μ and $\varepsilon_p(f_0)$. Therefore, using the usual procedure, we can write down the general solution of (1.1) in the following form:

$$X(p) = -\left[\frac{1}{T}\frac{\partial T}{\partial x_i}\,p_i\,A(p) + \Lambda_{ij}\,B_{ij}(p) + a + b_\mu\,p_\mu + c\,\varepsilon_p(f_0)\right] \qquad (3.1)$$

where A_i and B_{ij} satisfy the integral equations:

$$L(A_i) = f_0(p)\,p_i\,R(p), \quad \text{with} \quad A_i = p_i\,A(p), \qquad (3.2)$$

$$L(B_{ij}) = f_0(p)\,S_{ij}(p), \quad \text{with} \quad B_{ij} = \frac{\beta}{m}\left(p_i\,p_j - \tfrac{1}{3}p^2\,\delta_{ij}\right)B_i(p) + \delta_{ij}\,B_2(p). \qquad (3.3)$$

The Eqs. (2.3) and (3.3) are clearly solvable, since

$$\int \psi_i(p)\,p_i\,R(p)\,f_0(p)\,dp = \int \psi_i(p)\,S_{ij}(p)\,f_0(p)\,dp = 0 \qquad (3.4)$$

[6] A. K. Mitra and Sunanda Mitra, The Boltzmann-Landau Transort Equation II, Proc. Cambridge Phil. Soc. **64**, 189 [1968].

We have now to determine the five constants a, b_μ, c in (3.1). This we shall do by using the subsidiary conditions [1], (2.7a):

$$\int f_0(p)\, X(p)\, \varphi_i(p)\, dp = 0 \quad (3.5a) \qquad \text{where} \qquad \varphi_i(p) = R^{-1}\, \psi_i(p) \quad (3.5b)$$

Now since, according to [1], (2.5)

$$\varphi_i(p) = \psi_i(p) + \sum_{s=1}^{\infty} (-\beta)^s \int \widetilde{F}(p, q, f_0) \dots \widetilde{F}(q_{s-1}, q_s, f_0)\, \psi_i(q_s) \prod_{i=1}^{s} (f(q)\, dq_i) \quad (3.5c)$$

it is easy to see that: for $\psi_i = 1$, $\psi_1(p) = \varphi_1(p)$, for $\psi_i = p_i$ $(1 \leqq i \leqq 3)$, $\varphi_i(p) = p_i\, \varphi_2(p)$, and for $\psi_i = \varepsilon_p$, $\varphi_3(P) = \varphi_3(p)$. Then, as in ref. [6], we can use the subsidiary conditions to obtain

$$\int f_0 [\Lambda_{ij}\, \delta_{ij}\, B_2 + a + c\, \varepsilon_p(f_0)]\, \varphi_1(p)\, dp = 0\,, \quad (3.6a)$$

$$\int f_0(p) \left[\frac{1}{T} \frac{\partial T}{\partial x_\mu} A + b_\mu\right] \varphi_2(p)\, p^2\, dp = 0 \quad (3.6b)$$

and

$$\int f_0(p)\, [\Lambda_{ij}\, \delta_{ij}\, B_2 + a + c\, \varepsilon_p(f_0)]\, \varphi_3(p)\, dp = 0\,. \quad (3.6c)$$

Here we have used $\int (p_i\, p_j - \frac{1}{3} p^2\, \delta_{ij})\, G(p)\, dp = 0$, where G is a function of p alone. As usual [6], (3.6b) shows that $c_\mu \sim (1/T)\, \partial T/\partial x_\mu$ and the term $b_\mu\, p_\mu$ can be absorbed in the first term in (3.1) and $b_\mu = 0$, so that (3.6b) yields the subsidiary condition

$$\int A(p)\, \varphi_2(p)\, f_0(p)\, p^2\, dp = 0\,. \quad (3.8a)$$

Similarly, (3.6a) and (3.6c) show that a and c are proportional to $\Lambda_{ij}\, \delta_{ij}$ and therefore the terms involving a and c can be absorbed in the second term of (3.1) by redefining B_2 which thus satisfies the subsidiary conditions:

$$\int f_0(p)\, B_2(p)\, \varphi_1(p)\, dp = \int f_0(p)\, B_2(p)\, \varphi_3(p)\, dp = 0\,. \quad (3.8b)$$

(3.8a, b) are the new versions of the subsidiary conditions (2.10a, b) of Ref. [6]. Thus

$$X(p) = -\left[\frac{1}{T} \frac{\partial T}{\partial x_i} p_i\, A(p) + \Lambda_{ij}\, B_{ij}(p)\right] \quad (3.9)$$

where A_i and B_{ij} satisfy the symmetric linear integral Eqs. (3.2) and (3.3) satisfying the subsidiary conditions (2.10a, b) of Ref. [6].

4. The Effect of Ternary Collision on the First Order Density Corrections to Transport Coefficients

The first density corrections to η and λ already obtained in the binary collision approximation [6] will be extended here to include the ternary collision processes. However the previous calculations of [3] and [6] will be used whenever necessary in order to avoid repetitions.

(A) *Coefficient of viscosity* (η)

From the non-diagonal elements of the pressure tensor $\pi_{\mu\nu}$ given by (2.7) and from (3.9) we have

$$\pi_{\mu\nu} = -\frac{1}{m\, T} \frac{\partial T}{\partial x_\lambda} \int p_\mu\, p_\nu\, S_1(p)\, f_0(p)\, p_\lambda\, A(p)\, dp - \frac{1}{m} \Lambda_{ij} \int p_\mu\, p_\nu\, S_1(p)\, f_0(p)\, B_{ij}(p)\, dp \quad (4.1)$$

The usual argument [6] shows that the term involving $\partial T/\partial x_\mu$ in (4.1) vanishes. Thus, for $\mu \neq \nu$,

$$\pi_{\mu\nu} = -\frac{\Lambda_{ij}}{m} \int p_\mu\, p_\nu\, S_1(p)\, f_0(p) \left[\frac{\beta}{m} (p_i\, p_j - \tfrac{1}{3} p^2\, \delta_{ij})\, B_1(p)\, dp + \delta_{ij}\, B_2(p)\right]. \quad (4.2)$$

It is clear that, in the sum over i and j, the r.h.s. of (4.2) will vanish unless $i = \mu$, $j = \nu$, so that $\pi_{\mu\nu} = -2\, \eta\, \Lambda_{\mu\nu}$, where the coefficent of viscosity η is given by

$$\eta = \frac{\beta}{2\, m^2} \int p_\mu^2\, p_\nu^2\, S_1(p)\, f_0(p)\, B_1(p)\, dp = \frac{\beta}{30\, m} \int p^4\, S_1(p)\, f_0(p)\, B_1(p)\, dp \quad (4.3)$$

where we have used

$$\int G(p)\, p_\mu^2\, p_\nu^2\, dp = \frac{1}{15} \int G(p)\, p^4\, dp\,,$$

for any scalar function $G(p)$. (4.3) looks much simpler than given earlier [6]. Of course the B_{ij} here satisfy essentially a different integral equation (a symmetric one) and are subjected to subsidiary conditions (3.8b) different from those obtained earlier [6]. We now proceed to find the first order density correction to η directly from (4.3). As before [6], we take

$$B_1(p) = \frac{B_1{}^0(p) + n\, B_1{}^1(p) + \dots}{n}\,. \quad (4.4)$$

Also

$$f_0(q)/\int f_0(q)\,dq = \omega_0(q)\,[1+n\,\Theta(q)] \quad (4.5a)$$

where $\Theta(q) = 2[B(T) - F_{2,0}(p)]$ (4.5b)

Defining $F_0(p) = F_{2,0}(p)$, we have

$$F_0(p) = \int F_2(p,q)\,\omega_0(q)\,dq\,. \quad (4.5c)$$

(hereafter the index "0" will correspond to $n=0$). Here we notice that in (4.5a), $F_3(p_1, p_2, p_3)$ does not contribute to the first order density correction to f_0. Now with

$$S_1(p) = 1 + n\,S_1{}^1, \quad S_1{}^1 = \frac{m}{p}\frac{\partial F_0}{\partial p}$$

(see ref. [6]) we have from (4.3):

$$\eta = \eta_0 + n\,\eta_1\,, \quad (4.6a)$$

where

$$\eta_0 = \frac{\beta}{30\,m^2}\int p^4 B_1{}^0(p)\,\omega_0(p)\,dp \quad (4.6b)$$

$$\eta_1 = \left(\frac{d\eta}{dn}\right)_{n=0} = \frac{\beta}{30\,m^2}\int p^4[B_1{}^1(p) + S_1{}^1(p)\,B_1{}^0(p) + B_1{}^0(p)\,\Theta(p)]\,\omega_0(p)\,dp\,. \quad (4.6c)$$

The contribution to η_1 of the ternary collision appears through $B_1{}^1(p)$, which in turn is determined from an integral equation containing triple collisions, as we shall see now from the density expansion of (3.3), where $L(X)$ can be written as, according to [1], (3.8) and [6], (3.9):

$$L(X) = -X(p)\,f_0(p)\sum(p) + \int K(p,q)\,X(q)\,dq = D\,f_0\,, \quad (4.7a)$$

$$K'(p,q;f_0) = \int [2\,\sigma(s\,q\,|\,r\,p;f_0)\,f_0(s) - \sigma(s\,t\,|\,q\,p;f_0)\,f_0(p)]\,ds\,dt\,, \quad (4.7b)$$

$$K(p,q) = f_0(q)\,K'(p,q,f_0)\,, \quad (4.7c)$$

$$\sum(p) = \int \sigma(s\,t\,|\,r\,p;f_0)\,f_0(r)\,ds\,dt\,dr = \int G(r,p;f_0)\,f_0(r)\,dr \quad (4.7d)$$

where

$$G(r,p) = \int \sigma(s\,t\,|\,r\,p;f_0)\,ds\,dt\,. \quad (4.7e)$$

Now

$$\Sigma(p) = n[\Sigma_0(p) + n\,\Sigma_1(p) + \ldots] \quad (4.8)$$

where Σ_0 and Σ_1 are given by [6], (3.9). Here $(dG/dn)_0$ appears, which has already been calculated by GROSSMANN [4]. Now using the notation $[T(p)]$ for $T(p_1') + T(p') - T(p_1) - T(p)$ we have, as in Ref. [5],

$$\sigma(s\,q\,|\,t\,p;f_0) = \delta[p]\,\delta[E]\,\sigma(f_0) \quad (4.9a)$$

where

$$\delta[E] = \delta(E_0)\,[1 - n\,F_0(p)] \quad (4.9b)$$

with $E_0 = [p^2/2\,m]$. (Here we have used te relation $\times\,\delta'(x) = -\delta(x)$ used under the integral sign.) Assuming now the virial expansion:

$$\hat{\sigma}(f_0) = \sigma_0 + \int \sigma_1(K)\,f_0(K)\,dK + \ldots = \hat{\sigma}_0 + n\langle\hat{\sigma}_1\rangle_0 + \ldots \quad (4.10)$$

(where the dependence of $\hat{\sigma}(f_0)$ on the four momentum variables will be understood), we get from (4.9a) and (4.10)

$$\sigma(s\,q\,|\,t\,p;f_0) = \sigma_0(s\,q\,|\,t\,p) + n\,\sigma_1(s\,q\,|\,t\,p) + \ldots \quad (4.11a)$$

where

$$\sigma_0(s\,q\,|\,t\,p) = \hat{\sigma}_0(s\,q\,|\,t\,p) \quad (4.11b)$$

and

$$\sigma_1(s\,q\,|\,t\,p) = \hat{\sigma}_0(s\,q\,|\,t\,p) + n[\langle\sigma_1(s\,q\,|\,t\,p\rangle_0 - \hat{\sigma}_0(s\,q\,|\,t\,p)\,F_0(p)]\,. \quad (4.11c)$$

The effect of ternary collisions appears here through $\sigma_1(s\,q\,|\,t\,p;k)$ or equivalently $\langle\sigma_1(s\,q\,|\,t\,p\rangle_0$. From (4.7b), (4.5a) and (4.11a) it follows that

$$K'(p,q) = n[K_0(p,q) + n\,K_1(p,q)] \quad (4.12a)$$

where

$$K_0(p,q) = \int [2\,\sigma_0(s\,q\,|\,r\,p)\,\omega_0(s) - \sigma_0(s\,t\,|\,q\,p)\,\omega_0(p)]\,ds\,dt, \quad (4.12b)$$

$$K_1(p,q) = \int [2\,\sigma_1(s\,q\,|\,r\,p)\,\omega_0(s) - \sigma_1(s\,t\,|\,q\,p)\,\omega_0(p)]\,ds\,dt + \int [2\,\sigma_0(s\,q\,|\,r\,p)\,\Theta(s) - \sigma_0(s\,t\,|\,q\,p)\,\Theta(p)]\,ds\,dt\,. \quad (4.12c)$$

Now assuming $X = \frac{1}{n}(X_0 + n\,X_1 + \ldots)$, we obtain from (4.7c) and (4.12a), to the first order in the density n:

$$\int K(p,q)\, X(q)\, \mathrm{d}q = n \int \omega_0(q)\, K_0(p,q)\, X_0(q)\, \mathrm{d}q + n^2 \int \omega_0(q) \{K_0(p,q)\, X_1(q) + X_0(q)\, [K_1(p,q) + K_0(p,q)\, \Theta(q)]\}\, \mathrm{d}q\,. \tag{4.13}$$

The right hand side of (4.7a), namely $D f_0$, has been calculated in [1]. It is however easy to verify that, in the first order in density n, $D f_0$ remains the same as given in section 3 of ref. [6]. Putting $X = B_{ij}$, i.e., $X_0 = B^0_{ij}$, $X_1 = B^1_{ij}$, we obtain

$$I(B^0_{ij}) = -(\beta/m)\, \omega_0(p)\, (p_i p_j - \tfrac{1}{3} p^2 \delta_{ij}) \tag{4.14a}$$

$$I(B^1_{ij}) = \omega_0 M_{ij} = \omega_0(p)\, [(p_i p_j - \tfrac{1}{3} p^2 \delta_{ij})\, M_1(p) + \delta_{ij} M_2(p)] \tag{4.14b}$$

with

$$\omega_0(p)\, M_{ij}(p) = \omega_0(p)\, [M^{(b)}_{ij} + M^{(t)}_{ij}] \tag{4.15a}$$

where $M^{(b)}_{ij}$ is the contribution due to binary collisions given in [6] and $M^{(t)}_{ij}$ is the contribution due to the ternary collisions due to the first term, denoted $K_1^{(t)}$, in $K_1(p,q)$ given by (4.12c):

$$\omega_0(p)\, M^{(t)}_{ij}(p) = -\int \omega_0(p)\, B^0_{ij}(p)\, K_1^{(t)}(p,q)\, \mathrm{d}q\,. \tag{4.15b}$$

Here I is the binary collision operator defined by

$$I(h) = \omega_0(p)\, \Sigma_0(p)\, h(p) - \int K_0(p,q)\, \omega_0(q)\, h(q)\, \mathrm{d}q \tag{4.16}$$

as discussed in [6]. $K_0(p,q)$ is just the Chapmann-Enskog kernel, and thus B^0_{ij} is known, whereas the equations for $B_1{}^1(p)$ and $B_2{}^1(p)$ may be obtained as in Ref. [6].

(B) *The coefficient of thermal conductivity* (λ)

According to (2.11) and (3.9) we have

$$Q_\mu = -\frac{1}{m} \int p_\mu\, \varepsilon_p(f_0)\, S_1(p)\, f_0(p) \cdot \left[\frac{1}{T} \frac{\partial T}{\partial x} p_\nu A(p) + \Lambda_{ij} B_{ij}(p)\right] \mathrm{d}p\,. \tag{4.17a}$$

The usual argument shows that the term containing Λ_{ij} in (4.17) vanishes, so that the coefficient of thermal conductivity is given by:

$$\lambda = +\frac{1}{mT} \int p_\mu p_\nu\, \varepsilon_p(f_0)\, S_1(p)\, f_0(p)\, A(p)\, \mathrm{d}p\,. \tag{4.17b}$$

Since $G(p) = \varepsilon_p \cdot S_1(p) \cdot f_0(p)\, A(p)$ is an even function of p, it follows that λ vanishes unless $\mu = \nu$, so that

$$\lambda = \frac{1}{3mT} \int p^2\, \varepsilon_p(f_0)\, S_1(p) \cdot f_0(p)\, A(p)\, \mathrm{d}p\,. \tag{4.18}$$

Now assuming, as in [6],

$$A_i(p) = \frac{1}{n} [A_i{}^0(p) + n A_i{}^1(p) + \ldots] \tag{4.19a}$$

and following the same procedure as in case (A), we get, $\lambda = \lambda_0 + n \lambda_1$,

where

$$\lambda_0 = \frac{1}{3mT} \int \frac{p^4}{2m} A^0(p)\, \omega_0(p)\, \mathrm{d}p\,, \tag{4.19b}$$

$$\lambda_1 = (\mathrm{d}\lambda/\mathrm{d}n)_{n=0} = \int \left[\frac{p^2}{2m} S_1{}^1(p) + \left(F_0(p) + \frac{p^2}{2m} \Theta(p)\right) A^0(p) + \frac{p^2}{2m} A^1(p)\right] p^2\, \omega_0(p)\, \mathrm{d}p\,. \tag{4.19c}$$

As in case (A), the integral equation for $A_i{}^0$ and $A_i{}^1$ can be calculated using (4.13) and

$$A_i{}^\alpha = p_i A^\alpha(p) \qquad (\alpha = 0, 1)\,.$$

The results [putting $X = p_i A(p)$ in (4.13)] are:

$$I(A_i{}^0) = -\omega_0(p) \left(\frac{\beta p^2}{2m} - \frac{5}{2}\right) p_i\,, \tag{4.20a}$$

$$I(A_i{}^1) = -\omega_0(p)\, \vartheta(p)\, p_i \tag{4.20b}$$

with

$$\omega_0(p)\, \vartheta(p)\, p_i = \omega_0(p)\, [\vartheta^{(b)}(p) + \vartheta^{(t)}(p)]\, p_i \tag{4.20c}$$

where $\Theta^{(b)}(p)$ and $\Theta^{(t)}(p)$ are contributions from the binary [6] and ternary collision processes, as in case (A):

$$\omega_0(p)\, \vartheta^{(t)}(p)\, p_i = \int \omega_0(q)\, q_i A^0(q)\, K_1{}^{(t)}(p,q)\, \mathrm{d}q. \tag{4.20d}$$

The subsidiary conditions in each order of density corrections can also be found by putting $\psi_\mu = p_\mu$ in (3.5c), so that

$$p_\mu \varphi_2(p) = p_\mu - n\beta \int q F_2(p,q)\, \omega_0(q)\, \mathrm{d}q\,. \tag{4.21a}$$

Multiplying throughout by p_μ and adding we get

$$\varphi_2(p) = 1 - n \varphi_1{}^1(p) \tag{4.21b}$$

where

$$\varphi_1{}^1(p) = \frac{\beta}{p^2} (p \cdot q)\, F_2(p,q)\, \omega_0(q)\, \mathrm{d}q\,. \tag{4.21c}$$

The subsidiary condition (3.8a) then takes the form:

$$\int A^0(p)\ \omega_0(p)\ p^2\ \mathrm{d}p = 0\,, \qquad (4.22a)$$

$$\int A^1(p)\ \omega_0(p)\ p^2\ \mathrm{d}p = \int [\varphi_2^{1}(p) - \Theta(p)]\ p^2 A_0(p)\omega_0(p)\ \mathrm{d}p. \qquad (4.22b)$$

Similar conditions can also be found for B_2^{0} and B_2^{1} from (3.8b).

5. Discussion

In the previous section we have considered the effect of triple colision only. The calculations for the higher collision processes and therefore the density expansion of the transport coefficients can be carried out similarly. In the above calculation we see that $F_3(p_1, p_2, p_3)$ does not appear *explicitly*, the whole contribution to the first order density correction appears through $\sigma_1(K)$ [and equivalently $K_1^{(t)}(p, q)$]. F_3 will appear explicitly in the second density correction. The higher corrections do not seem to bring any better insight into the theory for the present and so we have omitted them here. Rather, it will be of even more interest to check whether the contribution from the ternary collisions gives rise to a logarithmic divergence in the first density corrections to the transport coefficients of a gas of hard discs, as has already been found by many authors [7] from other methods. All methods to find the effect of ternary collisions have however until now been formal. To come to a definite conclusion regarding our method we have to know F_2 and $\sigma_1(K)$, which is even difficult in the model given in [1], (1.5), (1.6). Finally the assumption [(1.4), ref. [1]] on $F_\nu(p_1, \ldots, p_\nu)$, though realistic, may not be known to be valid in general, at least where a strong anisotropy in the quasi-particle energy ε_p in the momentum space is present. In this case $D f_0$ will contain a term proprotional to grad n and the corresponding functions in the solution of the integral equations may not depend only on $|p|$. These are open problems and we hope to get at least some insight into them through realistic models.

The authors wish to thank Dr. ROBERT L. POE for critical reading of the manuscript.

[7] See references [5–7] of Ref. [5].

Berichtigung:

Zu A. K. MITRA and S. MITRA, Multicollisions in the Linearized Boltzmann-Landau Transport Equation, Z. Naturforsch. **25 a**, 115 [1970].

Page 117. Eq. (2.2a) read:

$$L(X) = \int \sigma(f_0)\ f_0(P)\ f_0(P_1)\ [x(P_1') + X(P') - X(P_1) - X(P)]\ \mathrm{d}P_1'\ \mathrm{d}P_1\ \mathrm{d}P' = D f_0\,, \qquad (2.2a)$$

Eq. (2.5) read:

$$\varphi(P) = R^{-1}X(P) = X(P) + \sum_{s=1}^{\infty} (-\beta)^s \int \widetilde{F}(P, Q_1; f_0) \ldots \widetilde{F}(Q_{s-1}, Q_s; f_0)\ X(Q_s) \prod_{i=1}^{s} f_0(Q_i)\ \mathrm{d}Q_i\,. \qquad (2.5)$$

Eq. (2.6a) read:

$$+ \sum_s (-\beta)^s \int \widetilde{F}(P, Q_1; f_0) \ldots \widetilde{F}(Q_{s-1}, Q_s; f_0)\ X(Q_s)\psi(P)\ f_0(P) \prod_{i=1}^{s} (f_0(Q_i)\ \mathrm{d}Q_i)\ \mathrm{d}P\,. \qquad (2.6a)$$

Eq. (2.6b) read:

$$A = \int f_0(P)\ \psi(P)\ X(P)\ \mathrm{d}P + \sum(-\beta)^s \int \widetilde{F}(Q_s, Q_1; f_0) \ldots \widetilde{F}(Q_{s-1}, P; f_0)\ \psi(Q_s)\ f_0(P) X(P) \prod_{i=1}^{s} (f_0(Q_i)\ \mathrm{d}Q_i)\ \mathrm{d}P$$

$$= \int f_0(P)\ \psi(P)\ X(P)\ \mathrm{d}P + \sum(-\beta)^s \int \widetilde{F}(Q_s, Q_{s-1}; f_0) \ldots \widetilde{F}(Q_1, P; f_0)\ X(P)\psi(Q_s)\ f_0(P) \prod_{i=1}^{s} (f_0(Q_i)\ \mathrm{d}Q_i)\ \mathrm{d}P$$

$$= \int f_0(P)\ X(P)\ \mathrm{d}P\ \{\psi(P) + \sum(-\beta)^s \int \widetilde{F}(P, Q_1; f_0) \ldots \widetilde{F}(Q_{s-1}, Q_s; f_0)\ \psi(Q_s) \prod_{i=1}^{s} (f_0(Q_i)\ \mathrm{d}Q_i) \qquad (2.6b)$$

Page 120, Eq. (3.13b) read:

$$S_1(p) = \left(1 + \frac{n\,m}{p} \frac{\partial \hat{F}(p, f_0)}{\partial p}\right). \qquad (3.13b)$$

Zu A. K. MITRA and S. MITRA, The First Order Density Corrections Including Ternary Collisions in a Boltzman-Landau Gas, Z. Naturforsch. **25 a**, 121 [1970].

Department of Electrical Engineering, Texas Tech. University, Lubbock, Texas

Page 125. Eq. (4.17b) read:

$$\lambda = \frac{1}{m\,T} \int p_\mu\, p_\nu\, \varepsilon_p(f_0)\ S_1(p)\ f_0(p)\ A(p)\ \mathrm{d}p\,. \qquad (4.17b)$$

Page 125 under Eq. (4.20c) read: $\vartheta^{(b)}(p)$ and $\vartheta^{(t)}(p)$.